Int. J. Therm. Sci. (1999) 38, 375–383
© Elsevier, Paris

Modelling of the transfer processes in turbulent film flows considering the influence of the vapour phase

Reinhard Würfel, Wolfgang Fratzscher

Institut für Thermodynamik, Energietechnik und Strömungsmechanik, Fachbereich Verfahrenstechnik, Martin-Luther-Universität Halle-Wittenberg, 06099 Halle, Germany

(Received 30 July 1998, accepted 17 September 1998)

Abstract — The ideas of Prigogine and Naue are the basis of theoretical modelling. The momentum and energy transfer processes at the distance of the thermodynamic equilibrium are associated with the formation of dissipative motion structures. The existence of rotation movements is defined for describing turbulent fluid flow in addition to translation motion quantities. The calculation of the velocity fields follows from these balance equations for translation and rotation movements. The interaction is expressed by the use of special source terms. The balance equations of energy are determined in the same way. The heat flux which is caused by angular momentum is formulated by a balance of its own. This model is applied to the situation of a vertical film flow with parallel gas flow. Particular boundary conditions are used. The calculated velocity and temperature profile in the film show a physically understandable interpretation. The convective heat transfer inside the film can be described by this spin modelling. A comparison of the calculated local heat transfer coefficients shows a satisfactory correspondence with the experimental data of different authors. © Elsevier, Paris.

film flow / turbulence / dissipative structure / angular momentum / heat transfer / vapour influence / velocity profile / theoretical model / interface shear stress

Résumé — Modélisation des processus de transfert dans des écoulements en film turbulents avec influence de la phase vapeur. Les idées de Prigogine et Naue sont la base de la modélisation théorique. Les processus de transfert de quantité de mouvement et d'énergie dans le domaine de l'équilibre thermodynamique sont associés à la formation des structures dissipatives. Les mouvements de rotation associés sont utilisés pour décrire l'écoulement turbulent du fluide, en association avec la quantité de mouvement de translation. Les équations de bilan écrites pour ces mouvements permettent de calculer les champs de vitesse. L'interaction est exprimée en termes de sources. L'équation de l'énergie est exprimée de la même manière. Le flux de chaleur dû au moment angulaire est formulé par un bilan sur cette grandeur. Ce modèle est appliqué à un film vertical s'écoulant parallèlement à un gaz. Des conditions aux limites particulières sont utilisées. Les profils de vitesse et de température calculés dans le film peuvent être expliqués physiquement. Le transfert de chaleur convectif dans le film peut être décrit par cette modélisation de spin. La comparaison des coefficients de transferts locaux calculés avec ceux qui sont issus de résultats expérimentaux d'autres auteurs montre un accord satisfaisant. © Elsevier, Paris.

écoulement en film / turbulence / structures dissipatives / moment angulaire / transfert de chaleur / couplage vapeur / profil de vitesse / modélisation / contrainte de cisaillement interfaciale

Nomenclature

a	thermal diffusivity	$m^2{\cdot}s^{-1}$
c	integration constant	
c_p	specific heat capacity	$kJ{\cdot}kg^{-1}{\cdot}K^{-1}$
d	diameter of the tube	m
f	average turbulence parameter of the film	
g	acceleration of gravity	$m{\cdot}s^{-2}$
l	characteristical vortex radius	m
n	exponent in equation (33)	
p	statical pressure	Pa
q	heat flux	$W{\cdot}m^{-2}$
$S^{v_k}\,\rho^{-1}$	source term in equation (1)	$m{\cdot}s^{-2}$
$S^{\varphi_i}\,\rho^{-1}$	source term in equation (8)	$m^2{\cdot}s^{-2}$
$S^{T}\,\rho^{-1}$	source term in equation (26)	$W{\cdot}m^{-3}$
$S^{\varphi T}\,\rho^{-1}$	source term in equation (29)	$W{\cdot}m^{-2}$
t	time	s

---

[*] Correspondence and reprints.
Reinhard.Wuerfel @iw.uni halle.de

T	temperature	K
u	velocity of the film in x-direction	$\mathrm{m \cdot s^{-1}}$
$\widetilde{u}$	average velocity of the film in the x-direction	$\mathrm{m \cdot s^{-1}}$
v	velocity of the film in the y-direction	$\mathrm{m \cdot s^{-1}}$
x, y, z	space direction	m
$\overline{x}_\mathrm{D}$	mass quality of the vapour	

Greek symbols

α	heat transfer coefficient	$\mathrm{W \cdot m^{-2} \cdot K^{-1}}$
δ	film thickness	m
ε_{jJk}	tensor of third order	
ξ	friction coefficent	
λ	thermal conductivity	$\mathrm{W \cdot m^{-1} \cdot K^{-1}}$
ν	kinematic viscosity	$\mathrm{m^2 \cdot s^{-1}}$
ρ	density	$\mathrm{kg \cdot m^{-3}}$
τ	shear stress	$\mathrm{N \cdot m^{-2}}$
φ	real spin	$\mathrm{s^{-1}}$
φ^T	rotational change of temperature field	$\mathrm{K \cdot m^{-1}}$
χ	coefficient of structure formation	
ω	rotor of the translational velocity field	$\mathrm{s^{-1}}$
ω^T	average changing of the temperature field in equation (30)	$\mathrm{K \cdot m^{-1}}$

Dimensionless quantities

$Fr = \frac{\widetilde{u}^2}{g\,\delta}$ Froude number of the film

$Nu_\mathrm{F} = \frac{\alpha_\mathrm{F}\left(\frac{\nu_\mathrm{F}^2}{g}\right)^{\frac{1}{3}}}{\lambda_\mathrm{F}}$ Nusselt number of the film

$Re_\mathrm{D} = \frac{\widetilde{u}_\mathrm{G}\, d}{\nu_\mathrm{G}}$ Reynolds number of the vapour

$Re_\mathrm{F} = \frac{\overline{u}\,\delta_\mathrm{F}}{\nu_\mathrm{F}}$ Reynolds number of the film

$Pr_\mathrm{F} = \frac{\nu_\mathrm{F}}{a_\mathrm{F}}$ Prandtl number of the film

$\widetilde{T} = \frac{T - T_\mathrm{w}}{T_\mathrm{Ph} - T_\mathrm{w}}$ dimensionless temperature

$\widetilde{u} = \frac{u}{\overline{u}}$ dimensionless velocity

$\widetilde{y} = \frac{y}{\delta}$ dimensionless length

$\widetilde{\varphi} = \varphi\,\frac{\nu_\mathrm{F}}{g\,\delta}$ dimensionless spin

$\widetilde{\varphi}^T = \varphi^T\,\frac{\delta}{(T_\mathrm{Ph} - T_\mathrm{w})}$ dimensionless changing of the temperature field

$\widetilde{\tau} = \tau\,\frac{1}{\rho_\mathrm{F}\, g\,\delta}$ dimensionless shear stress

Indices

F	film
G	gas, vapour
i	inside
j, k	space direction
lam	laminar
Ph	interface
RF	falling film
rot	rotational process
t, turb	turbulent
trans	translational process
w	wall
z	space direction
zp	two-phase flow

1. INTRODUCTION

The increase of heat and mass transfer in fluids depends on the behaviour of the turbulence. Therefore the calculation of the turbulence processes is necessary. The description of these flows is based on semiempirical and statistical turbulence theories [1, 2]. Special semiempirical theories can describe mass, momentum and heat transfers in multiphase systems, which are interesting for energy and chemical engineering. A general validity of modelling is not feasible today. The transfers of mass, momentum and energy in fluids are fundamentally irreversible processes because they are not in thermodynamic equilibrium [3]. In this area linear relations of thermal potentials and flows are not valid [4]. However such irreversible and stationary processes, and their combinations, can lead to new structures which are called dissipation structures, in contrast to the equilibrium structures of classic thermodynamics. The thermodynamics of non-equilibrium processes can describe the appearance and development of these structures on the bases of open systems which are characterized by their exchange of mass and energy with the environment [5]. Hence stationary non-equilibrium states with a higher level of order, which are stable at fluctuation, are produced. The formation of structures in irreversible processes is related to fixed conditions. At specific values of these conditions the structure of the systems can be changed qualitatively. A typical example is the transition from the laminar to the turbulent flow in fluids. Changing the flow state is a process of self organisation. A part of the system energy which is added to the thermal movement of the molecules, e.g. in the laminar flow, will be transferred to the macroscopically organised movement at the level of the new structure [6].

In a similar way, Naue and Bärwolff [7] described the appearance of self organisation processes and flow processes in structured systems. Most of the fluid movements are characterized by a development of structures, their specific length scales are longer than those at the molecular level. In addition to translation movement the angular momentum is also important.

Momentum transfer processes in laminar flows (so called homogenous momentum transfer) are accumulation, convection and induction processes at a molecular level. A general formulation of the momentum balance equation for the movement property v_k of a viscous fluid is [7]:

$$\frac{\partial v_k}{\partial t} + v_j \frac{\partial v_k}{\partial x_j} - \nu \frac{\partial^2 v_k}{\partial x_j^2} + \frac{1}{\rho} \frac{\partial p}{\partial x_k} - \frac{1}{\rho} S^{v_k} = 0 \qquad (1)$$

Density and viscosity are assumed to be constant. S^{v_k} is the source term. One of the possible sources is gravitational acceleration.

$$S^{v_k} = g_k$$

Rotational motions in the fluids, caused e.g. by vortex movements, have an influence on the source term and the translation process as well. Inhomogeneous transfer processes proceed above the molecular level. According to Naue and Bärwolff [7], they are a function of the direction and are characterised by turbulent vortex structures. The quantitative derivation of this consideration leads to differential equations of higher order for turbulent flows. The solution of these equation systems requires many experimental coefficients which are not available.

2. DESCRIPTION OF TURBULENT TRANSFER PROCESSES BY ANGULAR MOMENTA

The following model is also based on existing rotation velocities of turbulence vortexes, which are characterized in a real system at a field of rotation movement φ_i (spin field). For simplification it is assumed that the inhomogeneous momentum transfer processes can be described by balance equations as in equation (1). This way of describing requires the formulation of special source terms S. The characteristic spin φ_i is defined as the average value of the real rotational movements of different vortex structures, which are inside the fluid flow. These fields of spins cause forces which have influence on the translational movements.

The rotational movement and the force influences increase with increasing turbulence.

In a laminar fluid flow of Newtonian substances, a real spin φ_i does not exist. Here the energy dissipation takes place only in the translation processes. The friction between the fluid layers will be expressed by the shear stress equation

$$\frac{1}{\rho} \frac{\partial}{\partial x_j} \tau_{\mathrm{trans},jk} = \nu \frac{\partial^2 v_k}{\partial x_j^2} \qquad (2)$$

The average rotation ω_i of the translation velocity field is defined as the rotor [7]:

$$\omega_i = \frac{1}{2} \varepsilon_{ijk} \frac{\partial v_k}{\partial x_j} \quad \text{resp.} \quad \omega = \frac{1}{2} \operatorname{rot} v \qquad (3)$$

Then equation (2) can be transformed by applying the Laplace-operator in

$$\frac{1}{\rho} \frac{\partial}{\partial x_j} \tau_{\mathrm{trans},jk} = \nu \operatorname{grad} \operatorname{div} v - 2\nu \operatorname{rot} \omega \qquad (4)$$

Translation free conditions occur at the limit of high turbulent flow. The energy dissipation takes place only in rotational processes in vortex structures which are characterised by the spin field φ_i. Analogous to equation (4), a shear stress statement can be developed for the translation free flow with τ_{rot} as momentum flow in the rotational motion.

$$\frac{1}{\rho} \frac{\partial}{\partial x_j} \tau_{\mathrm{rot},jk} = -2\nu \operatorname{rot} \varphi \qquad (5)$$

Here the translation velocity gradient is inapplicable and the term ω_i is replaced by the real spin φ_i. In general, the energy dissipation in any turbulent fluid flow can be realised by translation and rotation processes. Thus the shear stress model resulting from the addition of equations (4) and (5) is:

$$\frac{1}{\rho} \frac{\partial}{\partial x_j} \tau_{jk} = \frac{1}{\rho} \frac{\partial}{\partial x_j} \tau_{\mathrm{trans},jk} + \frac{1}{\rho} \frac{\partial}{\partial x_j} \tau_{\mathrm{rot},jk}$$

$$\frac{1}{\rho} \frac{\partial}{\partial x_j} \tau_{jk} = \nu \frac{\partial^2 v_k}{\partial x_j^2} - 2\nu \operatorname{rot} \varphi \qquad (6)$$

Should equation (1)—the momentum balance equation for translation processes—be applied to turbulent flows, the source term must be written as follows:

$$\frac{1}{\rho} S^{v_k} = g_k - 2\nu \operatorname{rot} \varphi \qquad (7)$$

This equation expresses the influence of the real rotational motion of the vortex elements on the momentum transfer by translation. The angular momentum has an own balance equation which is independent of the momentum balance for the translation processes. This is the balance of the specific flow of the angular momentum where l^2 is the specific moment of inertia.

$$\frac{\partial (l^2 \varphi_i)}{\partial t} + v_j \frac{\partial (l^2 \varphi_i)}{\partial x_j} - \nu \frac{\partial^2 (l^2 \varphi_i)}{\partial x_j^2} - \frac{1}{\rho} S^{\varphi_i} = 0 \qquad (8)$$

The rotation processes will be influenced by fields of other properties; the field of the translation velocity is such an influence [7]. In this case, the source term S^{φ_i} can be determined by its relation to ω_i. A source effect occurs when the average rotation of the translation velocity field is different from the real angular motion φ_i. This relation can be quantified by introducing a dimensionless model parameter f, which is a scale factor for the average turbulence intensity within the investigated balance volume. The following equation for the source term is proposed

$$\frac{1}{\rho} S^{\varphi_i} = \nu \left(f \, \omega_i - \varphi_i \right) \tag{9}$$

Equation (9) allows a physically useful interpretation of the hydrodynamic limitation fields of the pure laminar and the high turbulence flows. In both cases the influence of the source term must be neglected ($S^{\varphi_i} \approx 0$). The limited value follows from:

$$\text{for} \quad \varphi_i \approx 0, \quad f \to 0 \qquad \text{with} \quad \omega_i \neq 0$$

$$\text{for} \quad \varphi_i >> 0, \quad f \, \omega_i \approx \varphi_i$$

The solution of the system of differential equations (1), (7), (8), (9) allows the description of momentum transfer in turbulent fluid flows. The calculation of the velocity profile requires the knowledge of the model parameters l^2 and f as well as the formulation of boundary relations for the fields of the real spin φ_i and the velocity of the translation v_k. A successful application of this method of describing turbulent flows in falling films with angular momentum is shown in [8]. The usefulness of a spin model should be discussed under the conditions of a 'real' turbulent two phase (gas–liquid) flow. The vertical turbulent film flow with vapour flow is used as an example. The most important resistance of mass and heat transfer is in the liquid film when phase change processes (condensation, absorption) with pure substances are applied. Therefore it is required to describe the transfer processes in the liquid phase. *Figure 1* shows the qualitative profile of the axial velocities in the x,y-area at vertical parallel flow of the phases. The significant deformation near the phase boundary is caused by the influence of the gas flow. The following balance equation system results from an incompressible, stationary turbulent film flow in the x,y-area of a volume element. The spin vector φ_z is given vertically in this area:

– mass balance equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}$$

– momentum balance equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{1}{\rho} \frac{\partial p}{\partial x} - \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + 2 \nu \frac{\partial \varphi_z}{\partial y} - g = 0$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{1}{\rho} \frac{\partial p}{\partial y} - \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - 2 \nu \frac{\partial \varphi_z}{\partial x} = 0 \tag{11}$$

– angular momentum balance equation

$$u \frac{\partial (l^2 \varphi_z)}{\partial x} + v \frac{\partial (l^2 \varphi_z)}{\partial y} - \nu \left(\frac{\partial^2 (l^2 \varphi_z)}{\partial x^2} + \frac{\partial^2 (l^2 \varphi_z)}{\partial y^2} \right) - \nu \left(f \, \omega_z - \varphi_z \right) = 0 \tag{12}$$

with $$\omega_z = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)$$

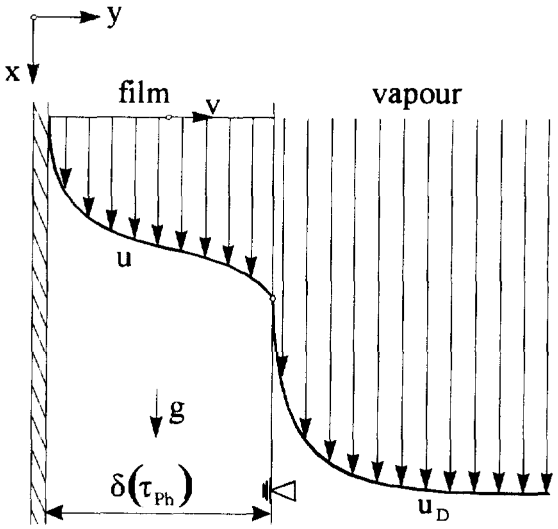

Figure 1. Qualitative velocity profile $u = u(y)$ in a vertical liquid film with parallel vapour flow.

3. APPLICATION OF THE ANGULAR MOMENTUM-TURBULENCE MODEL AT THE VERTICAL FILM FLOW

The coupled differential equation system equation (10), (11) and (12) can be solved principally by numerical methods. An analytical solution is possible with the following simplifications:

– a smooth, straight and developed film is assumed; pressure forces do not exist;

– the liquid phase is a Newtonian substance with constant values for ρ,ν and l^2 normal to the main flow direction;

– the changes in the main flow direction and the crosswise velocity will be neglected for the investigated volume element; the changes of the gas loading by phase transfer are small compared with the film loading

$$\left(\frac{\partial}{\partial x} \approx 0; \; v = 0 \right)$$

The simplified formulation of the equations (10), (11) and (12) can be united as shown in equation (13), where dimensionless properties and averaged values for the turbulence parameters f and l^2 about the cross-section will be introduced.

$$\frac{Fr}{Re_{\mathrm{F}}} \left(\frac{l}{\delta} \right)^2 \frac{\partial^4 \widetilde{u}}{\partial \widetilde{y}^4} - (f+1) \frac{Fr}{Re_{\mathrm{F}}} \frac{\partial^2 \widetilde{u}}{\partial \widetilde{y}^2} - 1 = 0 \tag{13}$$

The solution of equation (13) produces a relation to the velocity profile $\widetilde{u} = \widetilde{u}(\widetilde{y})$ across the film:

$$\frac{Fr}{Re_{\mathrm{F}}}\widetilde{u} = c_1 + c_2\,\widetilde{y} - \frac{\widetilde{y}^2}{2\,(f+1)} + c_3 \exp\left(-\sqrt{f+1}\,\frac{\delta}{l}\,\widetilde{y}\right) + c_4 \exp\left(\sqrt{f+1}\,\frac{\delta}{l}\,\widetilde{y}\right) \qquad (14)$$

where $c_1,\ldots,c_4$ are integration constants.

The relation to the dimensionless spin profile $\widetilde{\varphi}_z = \widetilde{\varphi}_z(\widetilde{y})$ across the film can also be derived:

$$\widetilde{\varphi}_z = \frac{1}{2}\,\frac{Fr}{Re_{\mathrm{F}}}\left(\frac{l}{\delta}\right)^2 \frac{\partial^3 \widetilde{u}}{\partial \widetilde{y}^3} - \frac{f}{2}\,\frac{Fr}{Re_{\mathrm{F}}}\,\frac{\partial \widetilde{u}}{\partial \widetilde{y}} \qquad (15)$$

The determination of the integration constants takes into consideration the boundary conditions for the film. The adhesive conditions at the wall for the translation velocity can be transferred to the angular movement. Therefore:

$$\widetilde{u}\big|_{\widetilde{y}=0} = 0 \qquad (16a)$$

$$\widetilde{\varphi}_z\big|_{\widetilde{y}=0} = 0 \qquad (16b)$$

are valid.

The influence of the gas flow is important for the condition at the gas–liquid interface. When the shear stresses in both phases are equal, we can write:

$$\widetilde{\tau}\big|_{\widetilde{y}=1} = \widetilde{\tau}_{\mathrm{Ph}} \qquad (16c)$$

For the flow in and around the tube the shear stress τ_{Ph} can usually be determined by equation (17):

$$\tau_{\mathrm{Ph}} = \xi_{\mathrm{Ph}}\,\frac{\rho_{\mathrm{D}}}{8}\,(u_{\mathrm{G}} - u)^2 \qquad (17)$$

where $u_{\mathrm{G}} >> u$ is usually valid.

The gas flow also influences the turbulence in the film and thus the angular movement of the vortex elements. It should be assumed that a correlation exists between the real angular momentum and the rotor of the velocity field for describing the deformation of the velocity profile near the phase boundary. Therefore a further boundary condition at the interface will be assumed to be

$$\widetilde{\varphi}_z\big|_{\widetilde{y}=1} = \operatorname{rot} u\big|_{\widetilde{y}=1} \qquad (16d)$$

Further information is necessary to determine the model parameters l^2 and f. l^2 will be fixed by the characteristic vortex diameter l of the turbulence structures. Assuming that the vortex elements in the film are arranged normal to the direction of the main stream, l can be determined by the addition of the diameters of the individual vortex elements.

$$l = \sum_{i=1}^{n} l_i = \frac{\delta}{2} \qquad (18)$$

For calculating the average turbulence parameter f, the integrated form of equation (14) will be used in the range $0 \le \widetilde{y} \le 1$ The result is expressed by equation (19):

$$\frac{Fr}{Re_{\mathrm{F}}} = \int_0^1 \left[c_1 + c_2\,\widetilde{y} - \frac{\widetilde{y}^2}{2\,(f+1)} + c_3 \exp\left(-\widetilde{y}\,\frac{\delta}{l}\,\sqrt{f+1}\right) + c_4 \exp\left(\widetilde{y}\,\frac{\delta}{l}\,\sqrt{f+1}\right)\right] \mathrm{d}\widetilde{y} \qquad (19)$$

Suitable equations for the film thickness δ at turbulent film flow with influence of the gas phase must be found. A theoretical description of the film flow using an entropy evaluation of momentum disproportion processes [9] allows the derivation of a model for the film thickness of a turbulent film flow. The equation is related to a critical film flow with neglectable vortex elements.

$$\delta_{\mathrm{RF,turb}} = \left[4\left(1 + \frac{e-1}{6\,e}\right)\right]^{\frac{-4}{4+\chi}} \left(\frac{3\,\nu^2}{g}\right)^{\frac{1}{3}} Re_{\mathrm{F}}^{\frac{1}{3}}\, Re_{\mathrm{F}}^{\frac{1}{4+\chi}} \qquad (20)$$

with $\chi = 1$ (overlapping of one dimensional vortex structures [9]).

Equation (20) can be extended to include the influence of flowing gases.

$$\delta_{\mathrm{turb}} = 0{,}304\left(\frac{3\,\nu^2}{g}\right)^{\frac{1}{3}} \left(1 + \frac{1}{2}\,\widetilde{\tau}_{\mathrm{Ph}}\right)^{-\frac{1}{3}} Re_{\mathrm{F}}^{\frac{8}{15}} \qquad (21)$$

The parameter f can be calculated by the coupling of equations (19) and (21). A modified Asali model [10] is used to calculate the interfacial friction coefficient ξ_{Ph} in equation (17). Different authors [11, 12] recommend this model for the condition of watery working fluids and for the flow inside tubes at normal pressure. A numerical comparison with other models [13, 14] resulted in useful values for the water-steam system in a broad field of phase loading [21].

Figure 2 shows the dimensionless velocity profile $\widetilde{u} = \widetilde{u}(\widetilde{y})$ in the film for different phase loading which is calculated from equation (14). The calculation is based on a vertical film flow around a tube with the working system ammonia/water ($\overline{x}_{\mathrm{D}} \approx 1$). The results are physically useful and show correctly, both qualitatively and quantitatively, the influence of the *Re*-numbers of the phases. For a parallel gas-flow the deformation of the profile near the phase boundary can be seen. The approached parabolic curve is also shown with liquid loading in the transition region.

Figure 3 shows the profiles of the dimensionless average angular movement of the vortex elements $\widetilde{\varphi}_z = \widetilde{\varphi}_z(\widetilde{y})$ which are calculated from equation (15). The important influence of the phase loadings can be seen in the spins which describe the turbulence in the whole region of the film. The different kind of the boundary areas can be explained. Furthermore the turbulent shear

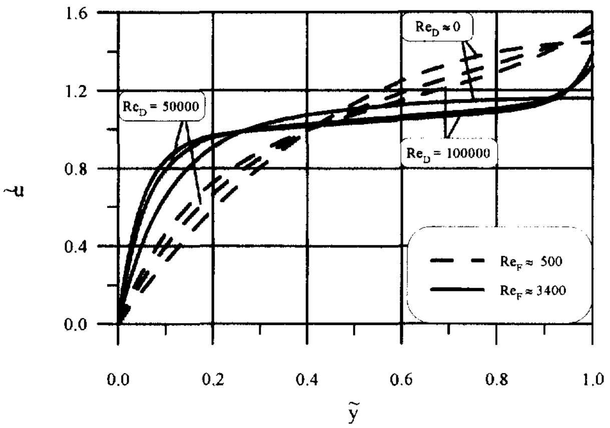

Figure 2. Calculated dimensionless velocity profiles (equation (14)) $\widetilde{u} = \widetilde{u}(\widetilde{y})$ in the turbulent liquid film depend on the phase loadings $(Re_{\mathrm{D}}, Re_{\mathrm{F}})$ working system: water/steam [21].

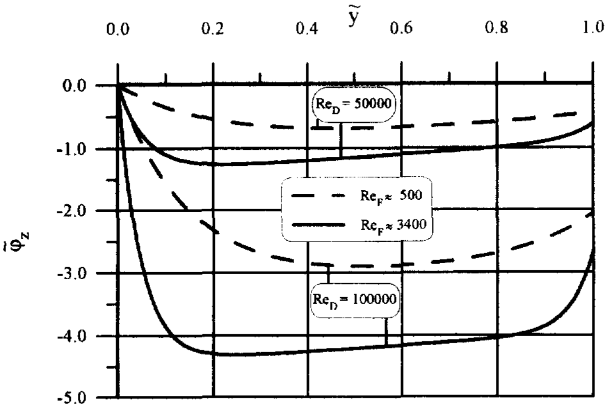

Figure 3. Calculated dimensionless spin profiles $\widetilde{\varphi}_z = \widetilde{\varphi}_z(\widetilde{y})$ for the vertical film flow (equation (15)) depend on the phase loadings $(Re_{\mathrm{D}}, Re_{\mathrm{F}})$; working system: water/steam [21].

stress is an interesting property for characterizing the hydrodynamics of a turbulent film flow. A great number of authors [15, 16] define an apparent turbulent viscosity ν_{t} analogously to the Newtonian shear stress hypothesis for the laminar flow:

$$\frac{1}{\rho}\frac{\partial}{\partial x_j}\tau_j = \frac{\partial}{\partial x_j}\left((\nu + \nu_{\mathrm{t}})\frac{\partial v_k}{\partial x_j}\right) \tag{22}$$

Formally, it is possible to separate the turbulent shear stress ν_{t}. For a film flow it can be written dimensionlessly by:

$$\widetilde{\tau}_{\mathrm{turb}} = \frac{\nu_{\mathrm{t}}}{\nu}\frac{Fr}{Re_{\mathrm{F}}}\frac{\partial \widetilde{u}}{\partial \widetilde{y}} \tag{23}$$

From equation (6) a turbulent shear stress can be derived on the basis of the turbulence model using the angular momentum balance.

For a film flow it follows that

$$\widetilde{\tau} = \frac{Fr}{Re_{\mathrm{F}}}\frac{\partial \widetilde{u}}{\partial \widetilde{y}} - 2\,\widetilde{\varphi}_z \tag{24}$$

and $\qquad \widetilde{\tau}_{\mathrm{turb}} = \widetilde{\tau}_{\mathrm{rot}} = -2\,\widetilde{\varphi}_z$

In *figure 4* the results concluded from equation (24) for different gas loadings are shown [21]. Results of other turbulence models which are calculated according to Mostofizadeh and Stephan [17], Limberg [15], Sandall [16], Mitrovic [18], Hubbard [19] and equation (23) are shown as well. The qualitative behaviour of the shear stress profiles is almost identical for all models in the central film region. There are, however, important differences at the boundary areas. This behaviour is based on the different assumptions regarding the laminar flow near the wall and the damping of the turbulence at the interface. These assumptions can be avoided by using the spin model. Therefore the values of shear stress $\widetilde{\tau}_{\mathrm{turb}}$ near the wall are comparatively high.

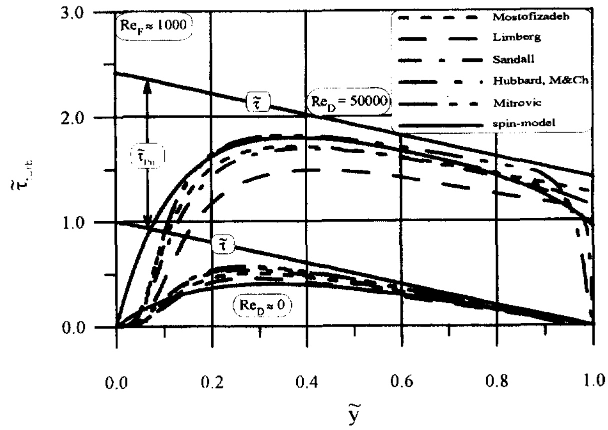

Figure 4. Calculated dimensionless profiles of the turbulent shear stress $\widetilde{\tau}_{\mathrm{turb}} = \widetilde{\tau}_{\mathrm{turb}}(\widetilde{y})$ (equation (24)) in comparison with models of different authors for the turbulent film flow.

4. HEAT TRANSFER IN TURBULENT FILM FLOW

The analytical description of the heat and mass transfer in turbulent film flows is not yet practicable because theoretical results for turbulent shear stresses, turbulent heat flow densities and diffusion flow densities, have been missing so far [20]. Furthermore the different analogies in momentum and heat transfer do not correspond correctly. A connection of heat and momentum transfer for a vertical film flow can be formulated using semiempirical turbulent transfer properties ν_{t} and λ_{t} [8].

$$1 + \frac{\lambda_{\mathrm{t}}}{\lambda} = A\left(1 + \frac{\nu_{\mathrm{t}}}{\nu}\right) Pr_{\mathrm{F}}^{n} \quad \text{for} \quad \frac{\nu_{\mathrm{t}}}{\nu} >> 1 \tag{25}$$

For equation (25) a most accurate calculation of the local heat transfer in the falling film flow is possibly using the formal determination of ν_t from the spin model (equations (23) and (24)). The temperature profiles near the film surface however could not be given correctly for a film flow which is influenced by a gas flow [21].

Another possibility of analogy considerations is given by transferring directly the concept of dissipative structures to turbulent heat transfer. It is thought that heat transfer by rotational motion of the vortex elements can be formulated by an independent balance equation. The heat flow related to the volume in the vertical film flow can be expressed in the following energy balance equation for the translation movement in the film

$$\rho\, c_p \left(\frac{\partial T}{\partial t} + v_j \frac{\partial T}{\partial x_j} \right) - \lambda \frac{\partial^2 T}{\partial x_j^2} - \frac{1}{\rho} S^T = 0 \qquad (26)$$

The source term $\frac{1}{\rho} S^T$ characterizes the influence of foreign fields. The turbulent part of the heat flow can be interpreted as that field which is caused by the angular movement of the vortex elements. An extension of the Fourier statement for the molecular heat flow with a term for the heat flow due to rotational processes, leads to:

$$\frac{\partial}{\partial x_j} \dot{q}_j = \frac{\partial}{\partial x_j} \dot{q}_{\mathrm{trans},j} + \frac{\partial}{\partial x_j} \dot{q}_{\mathrm{rot},j} \qquad (27)$$

or

$$\frac{\partial}{\partial x_j} \dot{q}_j = -\lambda \frac{\partial^2 T}{\partial x_j^2} - 2\,\lambda \operatorname{div} \varphi^T$$

in similar way to equations (3)–(6).

In equation (27) φ^T is the change of the temperature field which is caused by the rotation structures. These are a function of the direction. A formal extension of the conductive term in equation (26) leads to the source term in the formula:

$$\frac{1}{\rho} S^T = -2\,\lambda \operatorname{div} \varphi^T \qquad (28)$$

Under the condition of a laminar flow φ^T and—as a consequence—S^T do not apply. A further balance equation for the density of heat flow produced by the angular momentum of the vortex elements can be expressed as:

$$\rho\, c_p \left(\frac{\partial (l^2 \varphi_i^T)}{\partial t} + v_j \frac{\partial (l^2 \varphi_i^T)}{\partial x_j} \right) - \lambda \frac{\partial^2 (l^2 \varphi_i^T)}{\partial x_j^2} - \frac{1}{\rho} S^{\varphi^T} = 0 \qquad (29)$$

Analogously to equation (9) the source effect results from the difference of the translation and the rotational change of the temperature field.

$$\frac{1}{\rho} S^{\varphi^T} = \lambda \left(f \omega_i^T - \varphi_i^T \right) \qquad (30)$$

with $\omega^T = \frac{1}{2} \operatorname{grad} T$ (see [21]).

The turbulence parameter f is identical to its formulation in equation (9). The latter characterizes the connection of hydrodynamics and heat transfer. The coupling of equations (26) and (29) with the simplifications discussed above to the film flow can be expressed dimensionlessly as:

$$\left(\frac{l}{\delta} \right)^2 \frac{\partial^4 \widetilde{T}}{\partial \widetilde{y}^4} - (f+1) \frac{\partial^2 \widetilde{T}}{\partial \widetilde{y}^2} = 0 \qquad (31)$$

and

$$\widetilde{\varphi}_y^T = \frac{1}{2} \left(\frac{l}{\delta} \right)^2 \frac{\partial^3 \widetilde{T}}{\partial \widetilde{y}^3} - \frac{f}{2} \frac{\partial \widetilde{T}}{\partial \widetilde{y}}$$

The solution produces an equation for the calculation of the temperature field

$$\widetilde{\tau} = \widetilde{\tau}(\widetilde{y})$$

in the film:

$$\widetilde{T} = c_8 \frac{\widetilde{y}}{(f+1)} + c_5 + c_6 \exp\left(-\sqrt{f+1}\, \frac{\delta}{l}\, \widetilde{y} \right) + c_7 \exp\left(\sqrt{f+1}\, \frac{\delta}{l}\, \widetilde{y} \right) \qquad (32)$$

The required integration constants can be determined by the thermal boundary conditions both at the wall and at the interface. They result from the definition of the dimensionless temperature and from the assumption of a non turbulent heat transfer near the wall.

It follows:

$$\widetilde{T}\Big|_{\tilde{y}=0} = 0; \quad \widetilde{T}_{\tilde{y}=1} = 1; \quad \widetilde{\varphi}_y^T\Big|_{\tilde{y}=0} = 0$$

A maximum of the temperature field change, by the angular momentum, is assumed for the processes of condensation or absorption.

$$\frac{\partial \widetilde{\varphi}_y^T}{\partial \widetilde{y}}\Big|_{\tilde{y}=1} = 0$$

The connection between the constant c_8 and the axial local Nusselt number of the film is shown in [21]:

$$c_8 = \frac{Nu_{\mathrm{turb}}}{A\, Pr_{\mathrm{F}}^n} \qquad (33)$$

with $\alpha_{\mathrm{F,turb}} = \frac{Nu_{\mathrm{turb}}\, \lambda_{\mathrm{F}}}{\delta_{\mathrm{turb}}} = \frac{Nu_{\mathrm{F,turb}}\, \lambda_{\mathrm{F}}}{(\nu_{\mathrm{F}}^2/g)^{\frac{1}{3}}}$

The parameter A is used for a unified adaptation. This parameter is determined from comparison with different experimental data ($A = 0.55$). The exponent of the influence of the substances (Pr_{F}) is $n = 0.45$. Thus a description of the temperature profiles and the heat transfer in the film is facilitated by equations (32) and (33). For the whole area of the film flow the following average will usually be used:

$$Nu_{\mathrm{F}} = \sqrt{Nu_{\mathrm{F,lam}}^2 + Nu_{\mathrm{F,turb}}^2} \qquad (34)$$

Figure 5 shows calculated temperature profiles (see equation (32)) versus different phase loadings. The distribution of the transfer resistance is physically useful. The influence of the shear stress at the interface on the behaviour of the heat transfer is shown in *figure 6*. The efficiency of the spin model for describing the local heat transfer in a turbulent, falling film was investigated earlier including the conditions of condensation and absorption processes [8, 21, 22]. As a result, the experimental data of many authors can be reproduced satisfactorily over a broad range of substance properties and liquid phase loadings. *Figures 7* and *8* show some of the calculated values of the dimensionless heat transfer coefficients of the film flow which is influenced by gas flow. A comparison with experimental values of other authors [11, 23, 24, 25, 26] shows a satisfactory correspondence both for watery and organic working substances. *Figure 9* shows the differences of the values which are calculated with equations (32) and (34), and experimental film Nusselt numbers from different authors. The experimental conditions cover the following ranges:

$$140 < Re_F < 3\,500$$
$$6\,200 < Re_D < 3.6{\cdot}10^5$$
$$1.45 < Pr_F < 7.0$$

The maximum difference determined is about 30 %. This deviation is smaller when the $Nu_{F,lam}$ is calculated with the correction factor f_{well} which includes the waviness of the film.

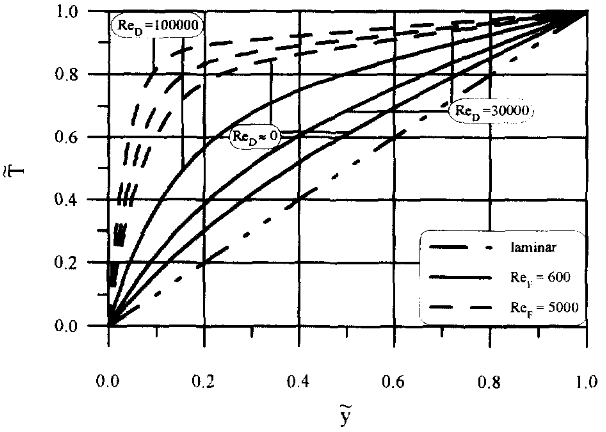

Figure 5. Calculated dimensionless temperature profiles $\tilde{T} = \tilde{T}(\tilde{y})$ in the turbulent liquid film at different vapour loading (Eq. 32); working system: water/steam.

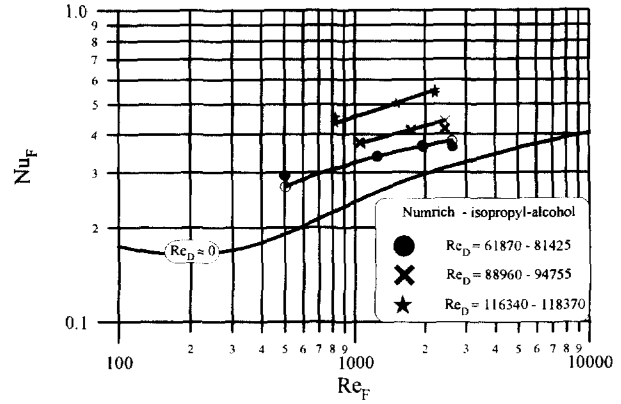

Figure 7. Comparison of calculated local Nusselt numbers (equations (32) and (34)) with experimental values [12] for film condensation at flowing vapour of iso-propyl-alcohol; $Pr_F \approx 7$, $d_i = 30$ mm.

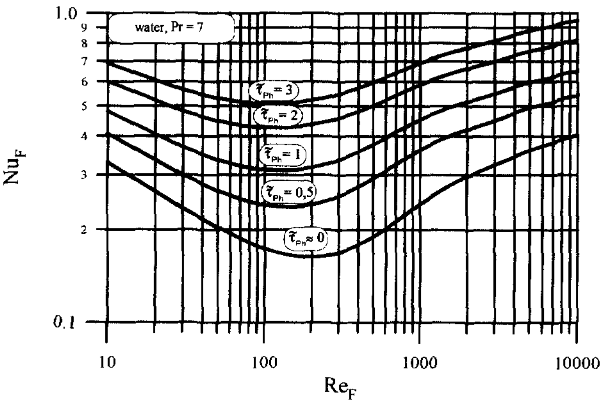

Figure 6. Calculated local Nusselt-numbers in the vertical liquid film depend on the liquid loading (Re_F) at different interfacial shear stresses $\tilde{\tau}_{ph}$ (equations (32) and (34)); working system: water/steam.

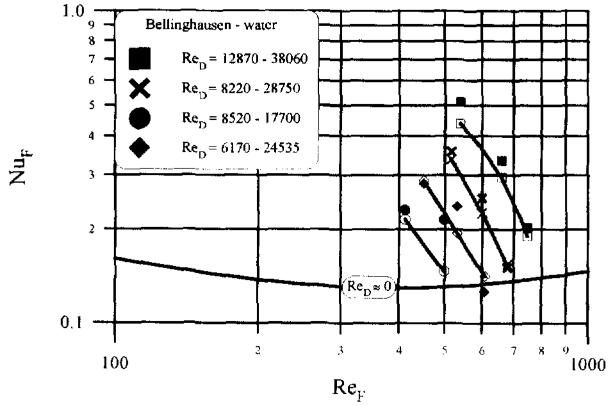

Figure 8. Comparison of calculated local Nusselt numbers (equations (32) and (34)) with experimental values [11] for film condensation of flowing steam; $Pr_F \approx 1.95$, $d_i = 14$ mm.

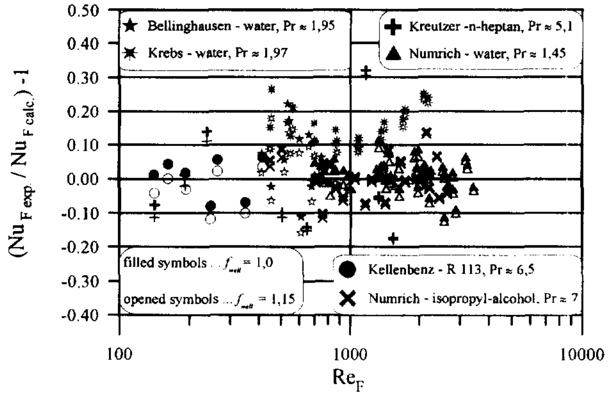

Figure 9. Differences between calculated Nusselt numbers (equations (32) and (34)) and experimental values of different authors for the film condensation of flowing vapours.

5. CONCLUSION

A theoretical turbulence model which includes angular momentum balances can be applied successfully to the description of velocity and temperature profiles in turbulent separate two phase flow. A satisfactory description of experimental data of the local heat transfer coefficient is possible for both the falling film flow and the turbulent film flow with parallel gas flow in a wide range of phase loading and physical properties. The application of semiempirical, turbulent transfer coefficients (ν_t, λ_t) is not required. The use of this model for other forms of two phase gas-liquid flow should be tested in further research work.

REFERENCES

[1] Prandtl L., Zeitschrift für angewandte Math. und Mechan. 5 (2) (1925) 136–139.

[2] Taylor G.I., Scientific Papers, G. K. Batchelor (Ed.), Vol. II, Cambridge Press, 1960.

[3] Baranowski B., Nichtgleichgewichtsthermodynamik in der physikalischen Chemie, VEB Deutscher Verlag für Grundstoffindustrie, Leipzig, 1975.

[4] Glansdorff P., Prigogine I., Thermodynamic theory of structure, stability and fluctuations, John Wiley and Sons, London, 1971.

[5] Ebeling W., Strukturbildung bei irreversiblen Prozessen, Teubner-Verlagsgesellschaft, Leipzig, 1976.

[6] Prigogine I., Stengers I., Dialog mit der Natur, Verlag Piper, München, 1990.

[7] Naue G., Bärwolff G., Transportprozesse in Fluiden, Deutscher Verlag für Grundstoffindustrie, Leipzig, 1992.

[8] Höhne J., Untersuchungen zum gekoppelten Stoff- und Wärmeübergang bei nichtadiabater Absorption im Film- und Blasenabsorber, VDI-Fortschrittsberichte, Reihe 3, Nr. 528, VDI-Verlag Düsseldorf, 1998.

[9] Höhne J., Würfel R., Fratzscher W., Allan F., Beschreibung einer Filmströmung durch Entropiebewertung, Chem. Technik, Leipzig 49 (3) (1997) 109–112.

[10] VDI-Wärmeatlas Abschnitt: Filmkondensation reiner Dämpfe, 7. Auflage, VDI- Verlag, Düsseldorf, 1994.

[11] Bellinghausen R., Die Filmkondensation bei hohen Dampfgeschwindigkeiten, Dissertation RWTH Aachen, Fakultät für Maschinenwesen, 1990.

[12] Numrich R., Stoff-, Wärme- und Impulsaustausch bei der Kondensation von Ein- und Mehrkomponentensystemen, Verlag Shaker, Aachen, 1994.

[13] Whalley P.B., Boiling, condensation and gas-liquid flow, Eng. Sci. Ser. 21, Clarendon Press Oxford, 1987.

[14] Andreussi P., Zanelli S., Downward annular and annular-mist flow, in: Durst F. (Ed.), Two phase momentum, heat and mass transfer in chemical process, Vol. 1, Hemisphere Publishing Corp., Washington, 1979.

[15] Limberg H., Int. J. Heat Mass Tran. 16 (9) (1973) 1691–1702

[16] Sandall O.C., AIChE. Symp. Ser. 80 (236) (1984) 3–9

[17] Mostofizadeh C., Stephan K., Wärme- und Stoffübertragung 15 (2) (1981) 93–115.

[18] Mitrovic J., Wärmeübergang bei der Filmkondensation reiner gesättigter Dämpfe an senkrechten Kühlflächen, VDI-Fortschrittsberichte Reihe 19, Nr. 22, VDI-Verlag Düsseldorf, 1987.

[19] Hubbard G.L., Mills A.F., Heat transfer across a turbulent falling film with cocurrent vapour flow, J. Heat Trans.-T. ASME 98 (2) (1976) 319–320.

[20] Baehr H.D., Stephan K., Wärme- und Stoffübertragung, Springer-Verlag, Berlin-Heidelberg, 1994.

[21] Allan F., Beschreibung der Transportvorgänge in turbulenten Filmströmungen unter Einfluß der Dampfphase, VDI-Fortschrittsberichte, Reihe 7, Nr. 338, VDI-Verlag, Düsseldorf, 1998.

[22] Höhne J., Allan F., Würfel R., Fratzscher W., Beschreibung des lokalen Impuls-, Stoff- und Wärmeüberganges in turbulenten Filmströmungen mit Hilfe von Spinfeldern, Chem. Technik, Leipzig 50 (1) (1998) 11–16

[23] Numrich R., Rennhack R., Hadley M., Einfluß der turbulenten Gasströmung auf den Wärmeübergang im Rieselfilm in einem vertikal durchströmten Kondensationsrohr, Abschlußbericht, DFG-Forschungsvorhaben Re 233/4, 1994.

[24] Krebs R., Kondensation von Dampf in Anwesenheit nichtkondensierbarer Gase in turbulent durchströmten senkrechten Kondensatorrohren, VDI-Fortschrittsberichte, Reihe 6, Nr. 153, VDI-Verlag Düsseldorf, 1984.

[25] Kellenbenz J., Wärmeübergang bei der Kondensation von strömenden Dämpfen reiner Stoffe und binärer Systeme, VDI-Fortschrittsberichte, Reihe 3, Nr. 365, VDI-Verlag Düsseldorf, 1994.

[26] Kreutzer T., Würfel R., Transportvorgänge bei turbulent durchströmten horizontalen und verschieden geneigten Kondensationsrohren, Abschlußbericht, DFG-Forschungsvorhaben Fr 1038/1-2, 1997.